ADDITIONAL NOTES

OF A

DISCUSSION OF TIDAL OBSERVATIONS

MADE IN CONNECTION WITH THE

COAST SURVEY AT CAT ISLAND,

LOUISIANA.

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Additional Notes of a Discussion of Tidal Observations made in connection with the Coast Survey at Cat Island, Louisiana; by Prof. A. D. Bache, Superintendent U. S. Coast Survey.*

In my communication on the subject of the tides at Cat Island, coast of Louisiana, at the New Haven meeting of the American Association,† I showed that I had succeeded in decomposing the curves of rise and fall into a diurnal and semidiurnal curve, which were nearly curves of sines; the diurnal curve having its maximum approximately nine hours in advance of the first maximum of the semidiurnal curve, and the interference of these two waves producing the tidal wave as observed. The comparison of the curves deduced from the observations for three months, and the computed curves of sines, was shown to be satisfactory. This comparison, made as before by averages of periods of a week combined into one general mean, has now been extended to the whole year, as shown in the subjoined table. By increasing the maximum ordinate of the diurnal curve 0·02 of a foot, which will make the rise and fall agree more nearly with the average deduced from observation, we obtain, as shown in No. 2, a resulting curve not differing in any ordinate more than a quarter of an inch from observation, and in which the positive and negative errors nearly balance, and the mean error deduced by summing the square of the errors is little more than one-eighth of an inch.

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* Read at the meeting of the American Association at Albany, and revised by the author, for publication in the American Journal of Science.

† See this Jour., xii, 341.

TABLE No. I.

Showing the comparison of diurnal and semidiurnal curves deduced from observations, with curves of sines. Diagram No. 1.

Hours from first mean level of water.	No. 1. FROM OBSERVATION.			No. 1. FROM CALCULATION.			Observations calculated No. 1.	No. 2. FROM CALCULATION.			Observations compared No. 2.
	Diurnal curve.	Semidiurnal curve.	Mean tidal curve.	Diurnal curve.	Semidiurnal curve.	Mean tidal curve.		Maximum ordinate diurnal curve increased 0·02 feet.	Semidiurnal as before.	Resulting mean tidal curves.	
	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
0	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	Same as foregoing.	0·00	0·00
1	·17	− ·03	·14	·15	− ·04	·11	·03	·15		·11	·02
2	·31	− ·06	·25	·28	− ·07	·21	·03	·29		·22	·02
3	·44	− ·08	·36	·40	− ·08	·32	·03	·42		·34	·02
4	·51	− ·06	·45	·50	− ·07	·43	·02	·51		·44	·00
5	·56	− ·03	·53	·55	− ·04	·51	·02	57		·53	·00
6	·57	− ·00	·57	·57	− ·00	·57	·00	·59		·59	− ·02
7	·56	+ ·03	·59	·55	+ ·04	·59	·00	·57		·61	− ·02
8	51	·06	·57	·50	·07	·57	·00	·51		·58	− ·01
9	·44	·08	·52	·40	·08	·48	·03	·42		·50	·02
10	·31	·06	·37	·28	·07	·35	·01	·29		·56	·00
11	·17	·03	·20	·15	·04	·19	.01	·15		·19	·00
12	·00	·00	·00	·00	·00	00	·00	·00		·00	·00

Nothing would be gained in closeness of representation of the result by displacing relatively the two tidal waves. It is only remarkable that in averages including the whole of the tides, even when most irregular, the results are so satisfactory. I have accordingly used the hypothesis of the representation of each wave by a curve of sines, deducing the maximum ordinate of computation from each observed ordinate. These laborious computations were made by Alexander S. Wadsworth, Jr., sub-assistant of the Coast Survey, and by Mr. P. B. Hooe. They give tables of heights of the diurnal and semidiurnal curve for each day of observation, which form the basis of the discussion of the heights. The next step after decomposing the curves of observation into diurnal and semidiurnal curves, is to discuss each separately to ascertain if they follow the laws deduced from them in regard to heights and times.

1. *Diurnal wave. Heights and times.*

If the diurnal curve is a curve of sines, then the ordinates found for each hour enable us to determine the value of the maximum or six-hour ordinate. Setting out from the mean line, then, we have for each day six determinations of the rise and fall above or below that line. Tables were computed from these, in which the daily curves were decomposed into their diurnal and semidiurnal components. In making these tables, the very irregular tides have been in general omitted. These tables were arranged according to the moon's declination, beginning and ending with the days on which the declination was zero, determining the maximum ordinate of each day from zero of declination. As the

irregular tides occur near the time of the moon's passing the equator, the averages of the heights about these times are deduced from a less number of observations than the others, and are therefore less reliable. The following table gives the average heights, with the number of days from which they have been deduced. No advantage resulted from displacing the epoch of the moon's declination relatively to the day of highest tide.

TABLE No. II. (Diagram 2.)

Showing the value of the maximum ordinates of the diurnal curve, on the several days from zero of declination of the moon to zero again, with the number of days from which the results are deduced.

Days from zero of declination.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of observations.														
Heights.	0·33	0·33	0·32	0·41	0·59	0·65	0·78	0·77	0·87	0·85	0·77	0·70	0·58	0·51
Nat. sin. 2 × moon's declin'n	0·05	0·11	0·12	0·24	0·41	0·46	0·52	0·58	0·60	0·59	0·54	0·46	0·37	0·27

The dependence of the height of the diurnal wave upon the moon's declination appears by comparing the lowest line of the table, containing the sine of twice the moon's declination, with the line next above it: it is also shown by the curves of Diagram No. 2. This agrees with Mr. Whewell's approximate formula for the diurnal inequality, namely, $dh = C. \sin 2\delta'$; in which dh is the difference in height of two consecutive high or low waters, C a constant, and δ' the moon's declination.

The variation of this same height with the sun's declination may be made at once apparent by classifying the heights for different values of the sun's declination with the same declination of the moon. The following table contains the greatest heights of the diurnal curves during the several lunations of the year, with the values of the sun's declination and of the moon's declination, grouped as described in the several columns.

TABLE No. III. (Diagram No. 3.)

Showing the effect of change of sun's declination on height.

Natural sine 2 sun's declination.	Number of lunations in group.	Natural sine 2 moon's declination.	Maximum ordinate diurnal curve.
Greater than 70°	5	·572	1·02
70 to 60	6	·577	0·99
60 to 40	6	·565	0·93
40 to 20	5	·530	0 94
20 to 00	4	·550	0·74

The effect of the change of parallax of the moon may be shown satisfactorily by grouping the values of the heights at the greatest southern declination of the moon, and for the greatest northern declination, for the year; comparing them for slightly varying declinations of the moon, for mean declinations of the sun, and for large variations of the parallax. The result is as shown in the following table, and in Diagram No. 4.

TABLE No. IV.

Showing the effect of change of moon's parallax on height.

Number of results.	Mean sine 2 moon's declination both series.	M'n sine 2 sun's declination both series.	M'n parallax correct. for 1st series.	M'n parallax correct. for 2d series.	Mean height for lesser parallax.	Mean height for greater parallax.
13½	59·4	48·5	52·9	65·9	0·74	0·88

The parallax correction is taken as the cube of the parallax multiplied by the sine of twice the moon's declination.

These are the principal variable terms in the formula derived by Mr. Lubbock, from Bernouilli's theory of the tides, for the diurnal inequality, namely,*

$$dh = \mathrm{B}[\mathrm{A} \,.\, \sin 2\delta \,.\, \cos(\psi-\varphi) + \sin 2\,\delta' \,.\, \cos\psi]\,;$$

in which dh is the difference in height of the morning and evening tide, B and A are constant coefficients, δ' is the moon's declination and δ the sun's; ψ is a small variable to be added to the mean lunitidal interval to give the interval corresponding to the moon's age, and φ is the hour angle of the moon at the time of transit. The second term, introducing the parallax of the moon, would be

$$m \,.\, \frac{\mathrm{P}'^{3}}{\mathrm{P}^{3}} \,.\, \sin 2\,\delta'\,;\dagger$$

in which m is a constant coefficient, P is the mean parallax, and P′ the parallax at the time under consideration.

In the application of this formula to the observations, the maximum ordinates, found as before stated, were tabulated; and first the coefficients were deduced from the cases corresponding to the maximum of the sine of twice the moon's declination and to the minimum of the sun's, and *vice versa*, neglecting the small variations due to $\cos(\psi-\varphi)$ and $\cos\psi$. This gave the following values for the coefficients, and the two sets of equations derived conformed with each other.

TABLE No. V.

Showing the value of coefficients deduced from maximum sine twice moon's declination and minimum of sun's, and vice versa; neglecting variations due to $\cos(\psi-\varphi)$ and $\cos\psi$.

	B. $\cos\psi$.	B . A . $\cos(\psi-\varphi)$.
First six months, ...	1·07	0·43
Second six months, ...	1·00	0·39
Whole year, ...	1·04	0·52

As each day's results are referred to the mean level of the day, and the mean of the low and high waters is taken as giving the height of the diurnal tide, the constant from the mean level of the whole should not appear in the values. In beginning these

* Transactions of the Royal Society of London, 1836, p. 223.

† Lubbock's Elementary Treatise on the Tides, London, 1839.

researches, I did not suppose that small differences would come out of them such as have been deduced. The reference to the level of each day compensated in a degree for the effect of an entire raising or depressing of the water by the wind's action.

The results promising success, the coefficients were deduced by the method of least squares for the first, and then for the second six months, and finally for the whole year. These laborious computations were made with much skill by Mr. W. W. Gordon, of the Coast Survey. The result for the second six months, in reference to the coefficient of the term of the sun's declination, is discrepant from the final result; but as the coefficients for the whole year were used, after endeavoring to trace the errors, if any, without immediate results, it was not pursued further.

TABLE No. VI.

Coefficients of $\cos(\psi-\varphi)$, *deduced from the method of least squares.*

	$B.\cos\psi$.	$B.A.\cos(\psi-\varphi)$.
First six months, ...	1·00	0·26
Second six months, ...	0·90	0·60
Whole year, ...	0·96	0·24

The sum of the positive and negative quantities balance, and rather better by the use of the coefficients from the first method, which differs chiefly in the coefficient of the sun's action.

The coefficient of the first term of dh is $B\times(A)$, and of the second term B; and it will be seen hereafter in discussing the semidiurnal tide, that (A) is 0·36, which, with $B=0·96$, gives $B\times(A)=0·34$.

A set of tables was next made, containing the values of the two terms of the formula for each day. To these was subsequently applied the small correction for the parallax from the term $\frac{P'^3}{P^3}$; and the terms, being summed, were compared with the observed maximum ordinate, and the difference in the final column of the table showed the residual to be accounted for.

For these tables I am indebted to Lieut. Trowbridge of the Corps of Engineers, assistant in the Coast Survey. The tabular quantities were also traced in curves, and then compared with the maximum ordinates. The positive and negative differences are usually small, not exceeding in the average about 0·12 of a foot, and are quite irregular.

The irregularities apparent in the phenomena themselves induced me, in first commencing this investigation, to hope merely to be able to trace the phenomena generally; but it now appears, from the character of the results obtained from the averages, that the theory may be followed much more closely by the results than I had at first supposed.

The accordance of observation and theory, after the corrections have been applied, is as good as the accidental errors of the separate results render necessary; as will be seen from the results for July given in the annexed table, and for July and part of August as given in Diagram No. 5: but as the averages seemed to indicate that the residuals would show the laws of the phenomena, I discussed them further.

TABLE No. VII.

Showing the value of maximum ordinates of the diurnal curve, computed from the moon's declination and parallax, and from the sun's declination, compared with ordinates from observation, for the month of July.

PART OF A TABLE FOR THE YEAR.

DAYS.	Maximum ordinate.	$0{\cdot}96 \cdot \frac{P'^3}{P^3} \cdot \sin 2\delta'$.	$0{\cdot}26 \cdot \sin 2\delta$.	
July 1	1·43	·66	·19	·60
2	·93	·59	·19	·17
3	·96	·35	·19	·43
4	·75	·33	·19	·26
5	·62	·22	·19	·17
6	·37	·08	·19	·10
7	·35	·14	·19	·12
8	·36	·14	·19	·03
9	·32	·25	·18	—·09
10	·52	·34	·18	·01
11	·65	·42	·18	·07
12	·75	·47	·18	·15
13	..	·54	·18	...
14	·78	·52	·18	·10
15	·73	·53	·18	·02
16	·62	·55	·18	—·10
17	·89	·53	·17	·19
18	·93	·48	·17	·29
19	·61	·40	·17	·06
20	·52	·15	·17	·20
21	·57	·00	·17	·40
22	·41	·02	·17	·22
23	·56	·33	·17	·07
24	·65	·45	·17	·05
25	·61	·55	·16	—·08
26	·77	·63	·16	·00
27	·90	·65	·16	·12
28	·86	·65	·16	·08
29	·90	·56	·16	·10
30	·90	·48	·16	·27
31	·69	·37	·15	·18

In looking for an explanation of the irregularities to the terms $(\psi - \varphi)$ and ψ, the residuals were classed according to the moon's age, and the averages taken for the separate hours. The result of these tables is given in that annexed, which shows the residual for each six months and for the year. I have introduced them for the half year, to show that the same law is deducible, notwithstanding the irregularities of the individual results, from the observations for each six months.

TABLE No. VIII. Diagram No. 6.

Showing the residuals from the comparison of computed and observed ordinates of diurnal curves, classed according to the ages of the moon.

Hours of moon's transit.	RESIDUALS.		
	First six months.	Second six months.	Mean.
0½	·23	·21	·22
1½	·17	·12	·13
2½	·15	·15	·15
3½	·15	·12	·13
4½	·16	·00	·08
5½	·08	—·03	·02
6½	·06	—·03	·01
7½	·08	—·02	·03
8½	·13	·04	·08
9½	·12	·12	·12
10½	·09	·14	·11
11½	·19	·14	·16

These residuals, instead of following the law of $\cos(\psi-\varphi)$, follow that of $\cos(2\psi-2\varphi)$, or that of the semidiurnal curve.

Before examining this result, which is shown in Diagram 6, I pass to the residual which results from carrying on the former table to 23½ hours; which was in fact the form of the table before the development of the law of variation showed that the term for 12½ hours belonged to 0½, instead of 11½, with which it would agree if the law of $\cos(\psi-\varphi)$ were followed. The following table contains the residuals in question, shown also in Diagram No. 7.

TABLE No. IX.

Showing residuals after deducting those following law of change of $\cos(2\psi-2\varphi)$.

Age of moon.	Residuals.	Residuals.	Mean.
hours.	feet.	hours.	feet.
0½	—·07	23½	—·01
1½	—·02	22½	—·01
2½	·01	21½	·01
3⅓	·03	20½	·03
4½	·00	19½	·02
5½	·01	18½	·04
6½	·05	17½	·08
7½	·04	16½	·09
8½	·07	15½	·04
9½	·02	14½	·00
10½	—·03	13½	·03
11½	·03	12½	·06
		Mean....	·03

The existence in the first residuals of the law belonging to the semidiurnal curve indicates that the separation of the two curves (diurnal and semidiurnal) is not complete, as indeed the hypothesis of a constant difference in time between the recurrence of the two maxima requires. Before undertaking to modify this hypothesis, I proceed to inquire whether these numbers would re-

ceive modification from any other source. In examining the hypothesis that the component curves were curves of sines, a separation of the several hourly ordinates was necessary, and thus the four points at which the curves for twenty-four hours cross the line of mean level were brought into consideration each day. Two of these points varied necessarily considerably in position, while the two twenty-four hours apart were regular. Having found that the curves of sines represent very nearly the observation, the law thus obtained may be used in computing from all the hourly observations of the day the values of the maximum ordinates for each curve; forming the ordinates of the observed curve into groups containing respectively the same positive and negative values of the ordinates of the diurnal curve, and again of the semidiurnal, arranging the groups for the consecutive twenty-four hours. It was soon apparent that the ordinates for the semidiurnal curve would in this way prove more considerable, in the average, than in the former mode of computation, and that the results would be more regular; that the ordinates of the diurnal curve would, on the average, be slightly diminished, and in general prove more regular. These revised tables have been prepared chiefly by Mr. W. W. Gordon and Mr. P. B. Hooe. They show on the average of the year a diminution of the maximum ordinates of the diurnal curve of 0·04 feet, and an increase of the maximum ordinates of the semidiurnal curve of 0·07 feet.

Classifying the corrections according to the moon's age, though they are irregular, it is apparent that there were entangled in the values of the former computed maximum ordinates, heights which belonged to the semidiurnal curve. The table of correction for the two periods of six months, and for the year, is given below.

TABLE No. X.

Showing the difference of maximum ordinates of diurnal curves, as computed by the last method of groups, and by that first applied.

Time of moon's transit.	Correction of maximum ordinates diurnal curve.		
	First 6 months.	Second 6 months.	Mean of year.
hours.	feet.	feet.	feet.
0½	−·10	−·06	−·08
1½	+·03	+·03	+·03
2½	−·08	−·02	−·05
3½	−·08	−·09	−·08
4½	−·08	−·09	−·08
5½	−·03	−·05	−·04
6½	−·02	−·02	−·02
7½	−·05	+·02	−·01
8½	+·01	+·02	+·01
9½	−·05	−·03	−·04
10½	−·08	−·03	−·05
11½	−·03	−·04	−·03

A consideration of the general formula for the height indicates a second correction. The height of high water, as given by the

formula is not the sum of the two greatest heights of the diurnal and semidiurnal tides. The hypothesis of the interference of the two waves makes the high water the sum of two ordinates (neither of which is the maximum), depending upon the laws of increase and decrease of the curves respectively, and of the relative position of the two ordinates. The correction due to this cause is readily found. The part of it which belongs to the diurnal curve will be the difference between D and D'. cos $(t-\mathrm{E})$; where E, according to the hypothesis of the interference of the two waves, is 9 hours; and t is the value for the maximum ordinate of the compound curve, namely (Proc. Amer. Assoc. Cambridge Meeting, page 289),

$$\operatorname{cosec} t - \sec t = \frac{4\mathrm{C}}{\mathrm{D}\sqrt{\frac{1}{2}}}.$$

This value of t, containing C (the maximum ordinate of the semidiurnal curve), shows that the quantity will vary with the time of the moon's transit, according to the half-monthly inequality of the height. Following the course which I have taken throughout this communication to give the resulting tables merely, I subjoin the corrections thus derived from the tables for $\frac{4\mathrm{C}}{\mathrm{D}\sqrt{\frac{1}{2}}}$ from observation, the computed values of t, and of $\mathrm{D}.\cos(t-\mathrm{E})$. The agreement of the general form of this correction with the theory is a new confirmation of the values of the quantities C and D, deduced from observation, which it contains.

TABLE No. XI.

Showing correction to height of the diurnal wave for difference of maximum ordinate, and of high water ordinate in compound curve.

Time of moon's transit.	Correction to maximum ordinate diurnal curve.
hours.	feet.
$0\frac{1}{2}$	—·03
$1\frac{1}{2}$	—·05
$2\frac{1}{2}$	—·03
$3\frac{1}{2}$	—·04
$4\frac{1}{2}$	—·04
$5\frac{1}{2}$	—·07
$6\frac{1}{2}$	—·08
$7\frac{1}{2}$	—·07
$8\frac{1}{2}$	—·06
$9\frac{1}{2}$	—·05
$10\frac{1}{2}$	—·05
$11\frac{1}{2}$	—·04

The correction furnished by the last two tables, and the corrected residual from the table, are given in Table No. 12 next following.

TABLE No. XII.

Showing residuals after correcting for new computations of ordinates, and difference between high water and maximum ordinates.

Time of moon's transit.	Correction of residual.	Residual.	Corrected residual.
hours.	feet.	feet.	feet.
$0\frac{1}{2}$	—·11	·22	·11
$1\frac{1}{2}$	—·02	·13	·11
$2\frac{1}{2}$	—·08	·15	·07
$3\frac{1}{2}$	—·12	·13	·01
$4\frac{1}{2}$	—·12	·08	—·04
$5\frac{1}{2}$	—·11	·02	—·08
$6\frac{1}{2}$	—·10	·01	—·08
$7\frac{1}{2}$	—·08	·03	—·05
$8\frac{1}{2}$	—·05	·08	·03
$9\frac{1}{2}$	—·09	·12	·03
$10\frac{1}{2}$	—·10	·11	·01
$11\frac{1}{2}$	—·07	·16	·09
			+·21
			Mean... ·017

Comparing the residuals in this table with the uncorrected ones, we find their magnitude much decreased; the average is now less than 0·02 of a foot: but the form of the series is, as before, that belonging to the semidiurnal curve, and is as well marked as when the quantities were more considerable. Diagram No. 6 shows this fact; containing the curve of residuals from Tables 8 and 12, and of half-monthly inequality deduced from the observations. This persistence in the form of the residuals affords the best evidence that the irregularities of the observations, and changes in the mode of computation, do not introduce errors of sufficient magnitude to mask the laws of the phenomena. I propose therefore to modify the original hypothesis, so as if possible to obliterate this form in the residual.

Some collateral questions have been examined in the course of this discussion, the results of which are interesting. One of these is the comparison of the maximum ordinates of the diurnal curve, corresponding to the moon's declination north and south. The average value of the sine of twice the moon's declination, and the corresponding average maximum ordinate for northern and southern declinations, are shown in the next table; from which it appears that if the values of $\sin 2\delta'$ were equal, the heights would not differ appreciably.

TABLE No. XIII.

Showing the mean value of twice the moon's declination, and the corresponding maximum ordinates for northern and southern declinations.

Sine $2\delta'$.	Maximum ordinate.	Sine $2\delta'$.	Maximum ordinate.
·410	·621	·351	·538
		·410	·531

Another question was, whether the residuals, of which Table No. 7 shows a part, contained any portion which varied with the moon's declination. To test this, the residuals for six months were grouped according to the declinations, with the following result.

TABLE No. XIV.

Containing the residuals after subtracting the terms containing the sine of twice the moon's declination, and the sine of twice the sun's declination, from the maximum ordinates, grouped according to the values of the sine of twice the moon's declination.

	Average value of twice sine moon's declination.				
Groups, ...	0 to 20	20 to 35	35 to 45	45 to 55	55 to 70
Average value, ...	·151	·147	·169	·115	·267
No. of observation, ..	(33)	(27)	(26)	(44)	(37)

The result indicates that there is no such term remaining in the residual.

Another question was, as to whether changing the epoch would improve the results. Several attempts of this kind were made at different stages of the work, but without any marked advantage. The average result for the year, as shown by comparing the dates of occurrence of the greatest and least maximum ordinate of the diurnal curve, and the greatest and least values of the term containing the moon's declination, is shown in the next table. The comparison is made in two different ways: first, by the date of the greatest value of the ordinate shown in the table of maximum ordinates; and secondly, by the date shown by the highest point of the curve, which was traced to represent the observations.

TABLE No. XV.

Showing results of comparison of dates of occurrence of the greatest and least maximum ordinate of the diurnal curve, and the greatest and least value of term containing the moon's declination.

DATE OF OCCURRENCE—AVERAGE IN DAYS.					
Maximum ordinate from table.	Maximum ordinate from curve.	Term embracing sun and moon's declination.	Minimum ordinate from table.	Minimum ordinate from curve.	Term containing sun and moon's declination.
15·4	16·1	16·0	16·5	16·6	16·0

The *times* of occurrence of the maximum of the diurnal curve are, as I have already stated, connected by the hypothesis with those of the semidiurnal curve. The times deducible from the observations were so irregular, that I supposed it impracticable to do more than this. Notwithstanding all these irregularities, it turns out that the laws of the phenomena for the times are deducible from the results. The average values follow those for the semidiurnal curve at the proper intervals. It will be practicable, therefore, to resume the examination of this part of the subject, which I accordingly purpose to do.

2. *Semidiurnal Curve.*

The results in relation to the semidiurnal curve have exceeded my anticipations. The half-monthly inequality, both in height and time, is very well shown by the maximum ordinates deduced; though the greatest value of the height is only 0·22 feet, and the irregularities in the separate observed high waters fall upon hours instead of minutes. In the following table, the maximum ordinates obtained by the method of groups are used, and the small correction for the difference between maximum and high water ordinates is omitted. The latter contains time of moon's transit corresponding to observed height; and the height computed from the formula given by Mr. Lubbock as resulting from Bernouilli's theory, and the difference between observation and theory.

TABLE No. XVI.

Showing half-monthly inequality in height.

Hours of moon's transit.	Observed height.	Computed height.	O – C. Diff. of observed and computed.
$0\frac{1}{2}$	·220	·223	— ·003
$1\frac{1}{2}$	·196	·206	— ·016
$2\frac{1}{2}$	·199	·174	·025
$3\frac{1}{2}$	·147	·131	·016
$4\frac{1}{2}$	·132	·087	·045
$5\frac{1}{2}$	·074	·056	·018
$6\frac{1}{2}$	·047	·056	— ·009
$7\frac{1}{2}$	·074	·087	— ·013
$8\frac{1}{2}$	·113	·131	— ·018
$9\frac{1}{2}$	·135	·174	— ·039
$10\frac{1}{2}$	·133	·206	— ·073
$11\frac{1}{2}$	·189	·223	— ·034

The greatest difference between observed and computed heights is 0·073, and the least difference 0·003; and the mean, without regard to sine, is 0·026. Diagram No. 8 shows the observed and computed curves of half-monthly inequality of heights. The average interval corresponds to $2^h\ 35^m$ of the moon's transit; which is therefore the zero point, or epoch of the half-monthly inequality in the interval.

The interval corresponding to the moon's

		h m		h m
transit at		3 30	is	11 45
"	for	9 30	"	13 05
			Diff. is	1 20

which, converted into arc, is 20°.

$$\text{Log tan } 20^\circ = \log (A) = 9{\cdot}56107;$$

$$(A) = 0{\cdot}364; \qquad \frac{1}{A} = 2{\cdot}747;$$

which is nearly the same as that obtained by Mr. Lubbock for Liv-

erpool. The difference between the greatest and least heights is

$$(0{\cdot}220-0{\cdot}047)=0{\cdot}173 \qquad \text{and } E=\frac{0{\cdot}173}{2(A)}=0{\cdot}238:$$

also the greatest height $0{\cdot}220=D+(E)\times(1+A)=D+{\cdot}325$; and $D=-0{\cdot}10$.

$$\text{Since } \frac{m'}{m'+M}=\frac{(0{\cdot}07480)^2}{(A)}=\frac{1}{65{\cdot}06}, \quad \frac{m'}{M}=\frac{1}{64{\cdot}06}.$$

For the half-monthly inequality of the intervals, we have

$$\tang 2\psi=\frac{(A)\times\sin 2\varphi}{1+(A)\times\cos 2\varphi}=\frac{0{\cdot}364\times\sin 2\varphi}{1+0{\cdot}364\times\cos 2\varphi};$$

and in the heights,

$$h=-0{\cdot}10+(E)\times(A)\times\cos(2\psi-2\varphi)+(E)\cos 2\psi$$
$$=-0{\cdot}10+0{\cdot}087\times\cos(2\psi-2\varphi)+0{\cdot}238\times\cos 2\psi.$$

The following table contains the half-monthly inequality of times deduced from the observations, and computed from the formula for tang 2ψ, and the comparison of observed and computed quantities.

TABLE No. XVII.

Showing differences between the results obtained from the observations and from formula.

Mean from observation 12h. 35m.

φ	ψ	C. From formula.	O. From observation.	O—C. +	O—C. —
h. m.	h. m.	h. m.	h. m.	m.	m.
0 30	0 08	12 27	12 31	04	
1 30	23	12 12	12 31	19	
2 30	36	11 59	11 19		40
3 30	42	11 53	11 45		08
4 30	38	11 57	12 03	06	
5 30	17	12 18	12 24	06	
6 30	17	12 52	12 38		14
7 30	38	13 13	13 09		04
8 30	42	13 17	13 27	10	
9 30	36	13 11	13 05		06
10 30	23	12 58	13 05	07	
11 30	08	12 43	13 05	22	
				+74	−72

$12^h\,35^m$ not being the exact mean of the observed times, the + and − differences do not balance exactly.

Diagram No. 9 shows the observed and computed results. The greatest and least heights correspond with the average interval, as they should do by Bernouilli's theory.

The average interval corresponds to $0^h\,23^m$ nearly, showing that transit E should be used instead of transit F.